

Two-Factor Between-Participants Designs

PSYC214: Statistics For Group Comparisons

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Week 7

PSYC214: Statistics for Group Comparisons
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2 x 2 Factorial Design
Structure
Main Effects
Simple Main Effects

Analysis a 2 x 2 Design
Fits
Basic Ratios
SS WITHIN BETWEEN A TOTAL
SS Main Effects
SS Interaction
F
ANOVA Table

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Single Main Effects Table

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Learning Objectives

- How to calculate *F* ratios for two-factor between-participants designs
- How to calculate simple main effects, if the interaction is significant

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Two-Factor Between-Participants Designs

- The simplest two-factor between-participants design is a 2 x 2 factorial design:
 - there are two factors, each with two levels, yielding a total of four cells or conditions
 - each participant contributes a single score to one condition only
- We can ask whether either of the **main effects** is significant
- We can also ask whether the **interaction** is significant
 - an interaction is interpreted in terms of the **simple main effects**

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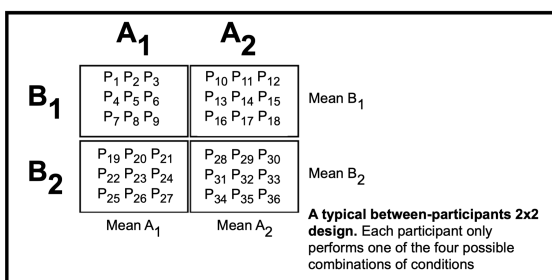
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A Typical Between-Participants 2 x 2 Design



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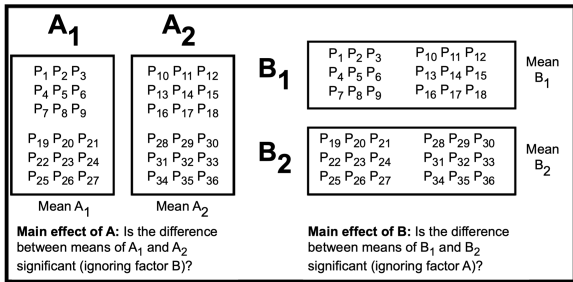
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Main Effects



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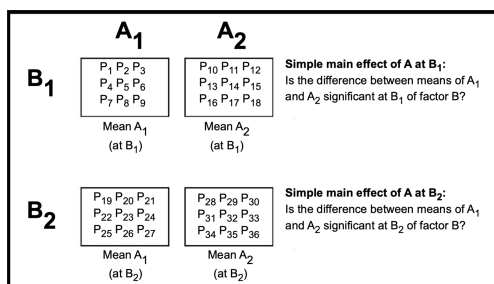
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Simple Main Effects of Factor A



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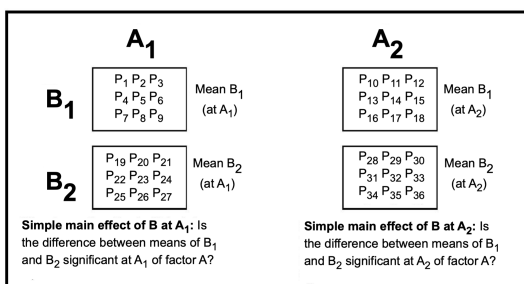
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Simple Main Effects of Factor B



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Simple Main Effects

- There are two ways a pair of simple main effects may differ in their trends:
 - one of a pair has a significant difference but not the other. For example, the mean of A₁ differs from the mean of A₂ at level B₂ but not at level B₁
 - both simple main effects are significant, but in the opposite direction. For example, the mean of A₁ is greater than the mean of A₂ at level B₁, but the pattern is reversed at level B₂

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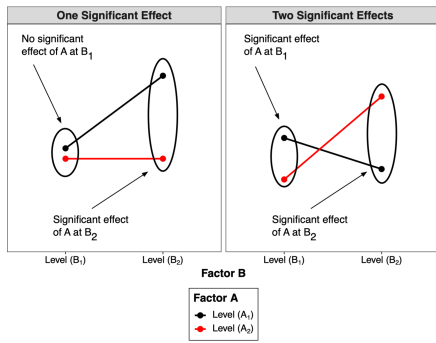
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Simple Main Effects



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Analysis a 2 × 2 Between-Participants Factorial Design

- The first stage of analysis seeks to uncover which of the two main effects and interactions are significant
- If the interaction is significant, then in a second stage we perform a simple main effects analysis
- Although a second factor has been added, the *F* ratio remains the same:

$$F = \frac{\text{treatment effects} + \text{experimental error}}{\text{experimental error}}$$

- As this is a between-participants design:

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

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Analysis a 2 × 2 Between-Participants Factorial Design

- The main difference is that there are now three *F* ratios, one for each of the three effects

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Hypothetical Data For COVID-19 Study

		Factor A: Fear	
		Level A ₁ no fear appeal	Level A ₂ fear appeal
Factor B: Level B ₁ no efficacy message	Efficacy	P ₁ 5	P ₁₃ 6
		P ₂ 4	P ₁₄ 4
		P ₃ 6	P ₁₅ 4
		P ₄ 4	P ₁₆ 5
		P ₅ 5	P ₁₇ 8
		P ₆ 6	P ₁₈ 3
Level B ₂ efficacy message		P ₇ 6	P ₁₉ 10
		P ₈ 6	P ₂₀ 9
		P ₉ 5	P ₂₁ 6
		P ₁₀ 3	P ₂₂ 9
		P ₁₁ 8	P ₂₃ 8
		P ₁₂ 3	P ₂₄ 7

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		Factor A: Fear		
		Level A ₁	Level A ₂	
		no fear appeal	fear appeal	Overall
Factor B:	Level B ₁ no efficacy message	5.00	5.00	5.00
Efficacy	Level B ₂ efficacy message	5.17	8.17	6.67
Overall		5.08	6.58	5.83

Navigation icons

Notes

Notation

$$SS_{BETWEEN} = \frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A} - \frac{(\sum Y)^2}{N}$$

$$SS_{WITHIN} = \sum Y^2 - \frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A}$$

$$SS_{TOTAL} = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

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Notation

$\frac{(\sum Y)^2}{N}$ is $\frac{(\text{grand total})^2}{\text{the number of scores that make up the grand total}}$

$\frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A}$ is $\frac{(\text{level total of } A_1)^2 + (\text{level total of } A_2)^2}{\text{the number of scores that make up each level}}$

$\sum Y^2$ is $\frac{(\text{score}_1)^2 + (\text{score}_2)^2 + (\text{score}_3)^2 (\text{and so on})}{1 (\text{only one number makes up each individual score})}$

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Basic Ratios

[T] : basic ratio of the grand total, $\frac{(\sum Y)^2}{N}$

[A] : basic ratio of the level totals, $\frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A}$

[Y] : basic ratio of the individual scores, $\sum Y^2$

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Basic Ratios

- To compute the components of a factorial between-participants ANOVA, two additional ratios are required
- [B] is the basic ratio of the level totals of factor B. If there are two levels in factor B, then [B] =

$$\frac{(\text{level total of } B_1)^2 + (\text{level total of } B_2)^2}{\text{the number of scores that make up each level}} = \frac{(\sum B_1)^2 + (\sum B_2)^2}{N_B}$$

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Basic Ratios

- $[AB]$ is the basic ratio of the cell totals, where a cell total is the total of all the scores in any one of the cells. For a 2×2 design, $[AB] =$

$$\frac{(\text{cell total of } A_1B_1)^2 + (\text{cell total of } A_1B_2)^2 + (\text{cell total of } A_2B_1)^2 + (\text{cell total of } A_2B_2)^2}{\text{the number of scores in each cell}}$$

$$= (\sum A_1B_1)^2 + (\sum A_1B_2)^2 + (\sum A_2B_1)^2 + (\sum A_2B_2)^2$$

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Calculating Basic Ratios For The Hypothetical Data

		Factor A: Fear		Total $B_1 =$ 30 + 30 = 60	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
		Level A_1 no fear appeal	Level A_2 fear appeal		
Factor B Efficacy	Level B_1 no efficacy message	Total $A_1B_1 =$ = 30	Total $A_2B_1 =$ = 30	Total $B_1 =$ 30 + 30 = 60	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
	Level B_2 efficacy message	Total $A_1B_2 =$ = 31	Total $A_2B_2 =$ = 49		
		Total $A_1 =$ 30 + 31 = 61	Total $A_2 =$ 30 + 49 = 79	$[Y] = 910$	
		$[A] = \frac{61^2 + 79^2}{12} = \frac{3721 + 6241}{12}$ $= \frac{9962}{12} = 830.1667$		$[7] = \frac{140}{24} = \frac{19600}{24} = 816.6667$	

$$[AB] = \frac{30^2 + 30^2 + 31^2 + 49^2}{6} = \frac{900 + 900 + 961 + 2401}{6} = \frac{5162}{6} = 860.3333$$

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Calculating Basic Ratios For The Hypothetical Data

		Factor A: Fear		Total $B_1 =$ $30 + 30 = 60$	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
		Level A_1 no fear appeal	Level A_2 fear appeal		
Factor B Efficacy	Level B_1 no efficacy message	Total $A_1B_1 =$ $= 30$	Total $A_2B_1 =$ $= 30$	Total $B_1 =$ $30 + 30 = 60$	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
	Level B_2 efficacy message	Total $A_1B_2 =$ $= 31$	Total $A_2B_2 =$ $= 49$		
		Total $A_1 =$ $30 + 31 = 61$	Total $A_2 =$ $30 + 49 = 79$	$[Y] = 910$	
		$[A] = \frac{61^2 + 79^2}{12} = \frac{3721 + 6241}{12}$ $= \frac{9962}{12} = 830.1667$		$[7] = \frac{140}{24} = \frac{19600}{24} = 816.6667$	

$$[AB] = \frac{30^2 + 30^2 + 31^2 + 49^2}{6} = \frac{900 + 900 + 961 + 2401}{6} = \frac{5162}{6} = 860.3333$$

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2 x 2 Factorial Design

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Analysis a 2 x 2 Design

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Calculating Basic Ratios For The Hypothetical Data

		Factor A: Fear		Total $B_1 =$ $30 + 30 = 60$	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
		Level A_1 no fear appeal	Level A_2 fear appeal		
Factor B Efficacy	Level B_1 no efficacy message	Total $A_1B_1 =$ $= 30$	Total $A_2B_1 =$ $= 30$	Total $B_1 =$ $30 + 30 = 60$	$[B] = \frac{60^2 + 80^2}{12}$ $= \frac{3600 + 6400}{12}$ $= 833.3333$
	Level B_2 efficacy message	Total $A_1B_2 =$ $= 31$	Total $A_2B_2 =$ $= 49$		
		Total $A_1 =$ $30 + 31 = 61$	Total $A_2 =$ $30 + 49 = 79$	$[Y] = 910$	
		$[A] = \frac{61^2 + 79^2}{12} = \frac{3721 + 6241}{12}$ $= \frac{9962}{12} = 830.1667$		$[7] = \frac{140}{24} = \frac{19600}{24} = 816.6667$	

$$[AB] = \frac{30^2 + 30^2 + 31^2 + 49^2}{6} = \frac{900 + 900 + 961 + 2401}{6} = \frac{5162}{6} = 860.3333$$

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Notes

Calculating The Sum of Squares For The Error Term

- Within-group variance is a measure of the extent to which people within each of the groups behave differently, despite being treated alike
- For a 2×2 between-participants design, people have been treated exactly alike *only* within each of the four cells
- To calculate the error term, we compute and combine the Sums of Squares and degrees of freedom using the smallest unit of identically treated participants—the four cells
- This gives a single measure of experimental error that can be used for calculating the F s for all the effects

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Notes

Calculating The Sum of Squares For The Error Term

- We calculate the error term, SS_{WITHIN} , as follows:

$$SS_{WITHIN} = [Y] - [AB] \quad SS_{WITHIN} \text{ will be designated } SS_{S/AB}$$

- This produces the error term that will be used to calculate all the F s
- This is the overall measure of the extent to which participants behaved differently despite being treated alike

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Notes

Between-Group Sum of Squares

- We also need to calculate the total between-group Sum of Squares for the four cells
- This is a measure of the variability due to the various experimental treatments
- It is a measure of how distant each of the four cell means is from the grand mean
- It tells us the overall extent to which the treatments caused scores to differ
- The between-group Sum of Squares is calculated as:

$$SS_{BETWEEN} = [AB] - [T] \quad SS_{BETWEEN} \text{ will be designated } SS_{AB}$$

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Notes

Total Sum of Squares

- We also need to calculate the total Sum of Squares
- This is a measure of total variability for the entire data set *irrespective* of experimental treatments
- It is calculated as:

$$SS_{TOTAL} = [Y] - [T]$$

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Calculating The Sums of Squares For The Two Main Effects

- Two between-group sums of squares are required, one for each of the main effects
- Each main effect is treated as being completely independent from the other
 - e.g., when calculating the main effect of factor A, the fact participants were treated in different ways at factor B is ignored
- The Sums of Squares for the two main effects are calculated as:

$$\begin{aligned} \text{for the between-group sums of squares for factor A, } SS_A &= [A] - [T] \\ \text{for the between-group sums of squares for factor B, } SS_B &= [B] - [T] \end{aligned}$$

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Calculating The Sums of Squares For The Two Main Effects

- To test the significance of the interaction, a final Sums of Squares is required
 - This is calculated as:
- $$SS_{INTERACTION}: SS_{A \times B} = [AB] - [A] - [B] + [T]$$
- This is the variability in the group means not accounted for by the main effects
 - It is the variability caused by the interaction between factor A and factor B

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Calculating The Sums of Squares Discussed So Far

$$\text{Within-group Sum of Squares: } SS_{S/AB} = [Y] - [AB]$$

$$= 910 - 860.3333 = 49.67$$

$$\text{Total between-group Sum of Squares: } SS_{AB} = [AB] - [T]$$

$$= 860.3333 - 816.6667 = 43.67$$

$$\text{Total Sum of Squares: } SS_{TOTAL} = [Y] - [T]$$

$$= 910 - 816.6667 = 93.33$$

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Calculating The Sums of Squares Discussed So Far

$$\text{Between-group Sum of Squares for factor A: } SS_A = [A] - [T]$$

$$= 830.1667 - 816.667 = 13.50$$

$$\text{Between-group Sum of Squares for factor B: } SS_B = [B] - [T]$$

$$= 833.3333 - 816.6667 = 16.67$$

$$\text{Sum of Squares for interaction: } SS_{A \times B} = [AB] - [A] - [B] + [T]$$

$$= 860.3333 - 830.1667 - 833.3333 + 816.6667 = 13.50$$

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Degrees of Freedom

- For the main effects:

$$df_A = (\text{number of levels in factor } A - 1) = (a - 1)$$

(*a* is the number of levels in factor *A*)

$$df_B = (\text{number of levels in factor } B - 1) = (b - 1)$$

(*b* is the number of levels in factor *B*)

- For the interaction:

$$df_{A \times B} = df_A \times df_B = (a - 1)(b - 1)$$

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Degrees of Freedom

- For the within-group variance (the error term):

$$df_{S/AB} = [(\text{number of cells} \times (\text{number of scores in cell} - 1))] = ab(s - 1)$$

(*s* is the number of scores in a cell)

- For the total degrees of freedom:

$$df_{TOTAL} = (\text{total number of scores} - 1) = (abs) - 1$$

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Degrees of Freedom

- The various degrees of freedom should add up so that:

$$df_{TOTAL} = df_A + df_B + df_{A \times B} + df_{S/AB}$$

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Calculating The Degrees of Freedom Discussed So Far

$$df_A = (a - 1) = 2 - 1 = 1 \text{ (factor A has two levels)}$$

$$df_B = (b - 1) = 2 - 1 = 1 \text{ (factor B has two levels)}$$

$$df_{A \times B} = (a - 1)(b - 1) = 1 \times 1 = 1$$

$$df_{S/AB} = ab(s - 1) = 2 \times 2(6 - 1) = 20 \text{ (six participants per cell)}$$

$$df_{TOTAL} = (abs) - 1 = (2 \times 2 \times 6) - 1 = 23$$

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Summary ANOVA Table By Components

Source	Sum of Squares	Degrees of freedom	Mean Square	F	p
A	$[A] - [T]$	$(a - 1)$	$\frac{[A] - [T]}{(a - 1)}$	Mean Square _A	tables
B	$[B] - [T]$	$(b - 1)$	$\frac{[B] - [T]}{(b - 1)}$	Mean Square _B	tables
A x B	$[AB] - [A] - [B] + [T]$	$(a - 1)(b - 1)$	$\frac{[AB] - [A] - [B] + [T]}{(a - 1)(b - 1)}$	Mean Square _{A x B}	tables
S/AB	$[Y] - [AB]$	$ab(s - 1)$	$\frac{[Y] - [AB]}{ab(s - 1)}$	Mean Square _{S/AB}	
TOTAL	$[Y] - [T]$	$(abs) - 1$			

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ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A	13.50	1			
B	16.67	1			
A x B	13.50	1			
S/AB	49.67	20			
TOTAL	93.33	23			

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ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	<i>F</i>	<i>P</i>
<i>A</i>	13.50	1	13.50		
<i>B</i>	16.67	1	16.67		
<i>A</i> × <i>B</i>	13.50	1	13.50		
<i>S/AB</i>	49.67	20	2.48		
<i>TOTAL</i>	93.33	23	4.06		

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ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	<i>F</i>	<i>P</i>
<i>A</i>	13.50	1	13.50	5.44	
<i>B</i>	16.67	1	16.67	6.72	
<i>A</i> × <i>B</i>	13.50	1	13.50	5.44	
<i>S/AB</i>	49.67	20	2.48		
<i>TOTAL</i>	93.33	23	4.06		

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ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	<i>F</i>	<i>P</i>
<i>A</i>	13.50	1	13.50	5.44	< .05
<i>B</i>	16.67	1	16.67	6.72	< .05
<i>A</i> × <i>B</i>	13.50	1	13.50	5.44	< .05
<i>S/AB</i>	49.67	20	2.48		
<i>TOTAL</i>	93.33	23	4.06		

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Source	Sum of Squares	Degrees of Freedom	Mean Square	<i>F</i>	<i>P</i>
<i>A</i>	13.50	1	13.50	5.44	< .05
<i>B</i>	16.67	1	16.67	6.72	< .05
<i>A</i> × <i>B</i>	13.50	1	13.50	5.44	< .05
<i>S/AB</i>	49.67	20	2.48		
<i>TOTAL</i>	93.33	23	4.06		

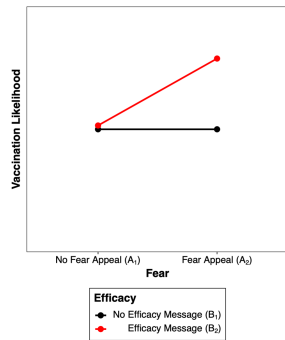
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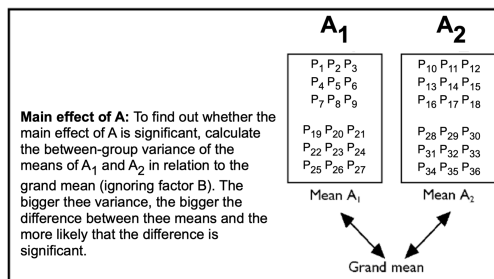
Simple Main Effects

- If the interaction is significant, then we interpret it by analysing the simple main effects
- In a 2 × 2 design, these are simply pairwise comparisons, analogous to using four *t*-tests
- This involves calculating the between-group variance for each simple main effect, before dividing each variance by the error term (*S*/*AB*) from the original ANOVA
- Thus, the significance of the simple main effects is evaluated using the same error term used to test the significance of the main effects and interaction

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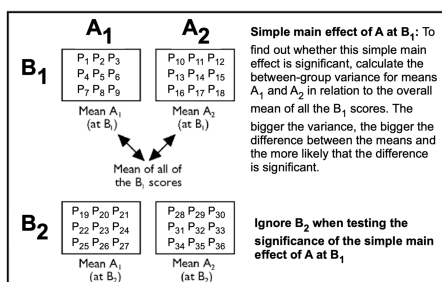
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Notes

Calculating Between-Group Sum of Squares

- The formula for calculating a between-group Sum of Squares is the basic ratio of the group totals of interest, minus the basic ratio of the total of these totals [7]
- For example, the formula for calculating the between-group variance for the main effect of factor A is [A] - [7]
- The basic ratios used to calculate the between-group variances for the simple main effects are analogous to these

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Notes

Calculating Between-Group Sum of Squares

- For example:
- $[A_{B_1}]$ is the basic ratio of factor A, but only for the B_1 scores: square the total for A_1B_1 , square the total for A_2B_1 , add the squares together and divide by the number of scores that make up each cell.
- $[T_{B_1}]$ is the basic ratio of the total of the scores at level B_1 of factor B: take the total of all the scores in level B_1 and square the total, divide the square by the number of scores making up this total.
- *Eight basic ratios are required to test the four simple main effects ...*

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Calculating Between-Group Sum of Squares

Sum of Squares between groups of factor A at level B_1 ($SS_{A \text{ at } B_1}$):
 $[A_{B_1}] - [T_{B_1}]$

Sum of Squares between groups of factor A at level B_2 ($SS_{A \text{ at } B_2}$):
 $[A_{B_2}] - [T_{B_2}]$

Sum of Squares between groups of factor B at level A_1 ($SS_{B \text{ at } A_1}$):
 $[B_{A_1}] - [T_{A_1}]$

Sum of Squares between groups of factor B at level A_2 ($SS_{B \text{ at } A_2}$):
 $[B_{A_2}] - [T_{A_2}]$

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Calculating Between-Group Degrees Of Freedom

- All degrees of freedom are equal to the number of ((number of levels in each simple main effect)) - 1
- For the two simple main effects of A, the degrees of freedom are given by $(a - 1)$, where a is the number of levels in factor A
- For the two simple main effects of B, the degrees of freedom are given by $(b - 1)$, where b is the number of levels in factor B

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Calculating Between-Group Sum of Squares

		Factor A: Fear		
		Level A ₁ no fear appeal	Level A ₂ fear appeal	
Factor B Efficacy	Level B ₁ no efficacy message	Total A ₁ B ₁ = 30	Total A ₂ B ₁ = 30	Total B ₁ = 30 + 30 = 60
	Level B ₂ efficacy message	Total A ₁ B ₂ = 31	Total A ₂ B ₂ = 49	Total B ₂ = 31 + 49 = 80
		Total A ₁ = 30 + 31 = 61	Total A ₂ = 30 + 49 = 79	

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Notes

Calculating Between-Group Sum of Squares

- Fear (no fear appeal vs. fear appeal) for no efficacy message (A at B₁)

$$[A_{B_1}] = \frac{30^2 + 30^2}{6} = 300 \quad [T_{B_1}] = \frac{60^2}{12} = 300 \quad [A_{B_1}] - [T_{B_1}] = 0$$

- Fear (no fear appeal vs. fear appeal) for efficacy message (A at B₂)

$$[A_{B_2}] = \frac{31^2 + 49^2}{6} = 560.33 \quad [T_{B_2}] = \frac{80^2}{12} = 533.33 \quad [A_{B_2}] - [T_{B_2}] = 27$$

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2 × 2 Factorial
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Calculating Between-Group Sum of Squares

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	Level B ₂ efficacy message	Total A ₁ B ₂ = 31	Total A ₂ B ₂ = 49	Total B ₂ = 31 + 49 = 80
		Total A ₁ = 30 + 31 = 61	Total A ₂ = 30 + 49 = 79	

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- Fear (no fear appeal vs. fear appeal) for efficacy message (A at B₂)

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- Efficacy (no efficacy message vs. efficacy message) for no fear appeal (B at A_1)

$$[B_{A_1}] = \frac{30^2 + 31^2}{6} = 310.17 \quad [T_{A_1}] = \frac{61^2}{12} = 310.08 \quad [B_{A_1}] - [T_{A_1}] = .09$$

- Efficacy (no efficacy message vs. efficacy message) for fear appeal (B at A_2)

$$[B_{A_2}] = \frac{30^2 + 49^2}{6} = 550.17 \quad [T_{A_2}] = \frac{79^2}{12} = 520.08 \quad [B_{A_2}] - [T_{A_2}] = 30.09$$

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Calculating Between-Group Sum of Squares

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Summary Simple Main Effects Table By Components

SOURCE	Sum of Squares	Degrees of freedom	Mean Square	F	p
A at B ₁	$[A_{B_1}] - [T_{B_1}]$	(a - 1)	$\frac{[A_{B_1}] - [T_{B_1}]}{(a - 1)}$	$\frac{\text{Mean Square}_{A \text{ at } B_1}}{\text{Mean Square}_{S/AB}}$	tables
A at B ₂	$[A_{B_2}] - [T_{B_2}]$	(a - 1)	$\frac{[A_{B_2}] - [T_{B_2}]}{(a - 1)}$	$\frac{\text{Mean Square}_{A \text{ at } B_2}}{\text{Mean Square}_{S/AB}}$	tables
B at A ₁	$[B_{A_1}] - [T_{A_1}]$	(b - 1)	$\frac{[B_{A_1}] - [T_{A_1}]}{(b - 1)}$	$\frac{\text{Mean Square}_{B \text{ at } A_1}}{\text{Mean Square}_{S/AB}}$	tables
B at A ₂	$[B_{A_2}] - [T_{A_2}]$	(b - 1)	$\frac{[B_{A_2}] - [T_{A_2}]}{(b - 1)}$	$\frac{\text{Mean Square}_{B \text{ at } A_2}}{\text{Mean Square}_{S/AB}}$	tables
S/AB	$[Y] - [AB]$	ab(s - 1)	$\frac{[Y] - [AB]}{ab(s - 1)}$		

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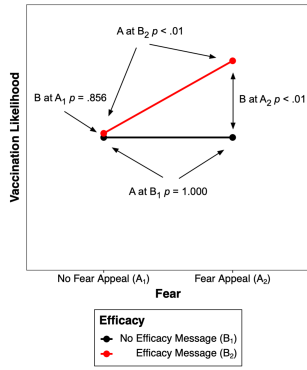
Simple Main Effects Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A at B ₁	0.00	1	0.00	0.00	1.000
A at B ₂	27.00	1	27.00	10.89	< .01
B at A ₁	0.09	1	0.09	0.04	.856
B at A ₂	30.09	1	30.09	12.13	< .01
S/AB (error)	49.67	20	2.48		

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Interaction Plot



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Additional Resources

- The R code for all plots generated in this lecture (minus annotations) has been uploaded with these slides to the Week 6 lecture folder (R Plots For Lecture 7.R)

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Notes

In Next Week's Lab ...

- Running a 2 × 2 (and 2 × 3) between-participants ANOVA in R
- Calculating and interpreting simple main effects

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References

Roberts, M. J., & Russo, R. (1999, Chapter 9–10). *A student's guide to Analysis of Variance*. Routledge: London.

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